Solution to MHT CET – 2021
20th September (Shift - 2)

Section I

PHYSICS

1. (B)

Since it is compressed isothermally, the temperature remains constant. The rms speed is given by,

$$c = \sqrt{\frac{3RT}{M}}$$

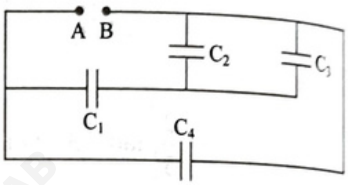
Since temperature T remains constant, the rms speed remains unchanged.

2. (D)

$C_1 = C_2 = C_3 = C_4 = 6\ \mu F$

C_2 and C_3 are in parallel.

Hence their equivalent capacitance, $C_5 = 2 \times 6 = 12\ \mu F$



C_5 and C_1 in series.

Hence their equivalent capacitance C_6 is given by

$$\frac{1}{C_6} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

$\therefore\ C_6 = 4\ \mu F$

C_6 and C_4 are in parallel.

Hence their equivalent capacitance is $C = 4 + 6 = 10\ \mu F$

3. (C)

Relation between P and V is given as

$PV^{3/2} = \text{constant}$

For adiabatic process, $PV^{\gamma} = \text{constant}$

$\therefore\ \gamma = \frac{3}{2}$

The relation between T and V is given by

$TV^{\gamma - 1} = \text{constant}$

$\therefore\ T_1V_1^{\gamma - 1} = T_0V_0^{\gamma - 1}$

$\therefore\ T_1V_1^{1/2} = T_0V_0^{1/2}$

$$\therefore\ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{1/2} = 2^{1/2} = \sqrt{2}$$

$\therefore\ T_2 = \sqrt{2}\,T_1 = \sqrt{2}\,T$

4. (D)

$\lambda_1 = 1\ \text{nm},\ \lambda_2 = 0.5\ \text{nm}$

$\therefore\ \lambda_2 = \frac{\lambda_1}{2}$

$\lambda = \frac{h}{p} \qquad \therefore\ P_2 = 2P_1$

Kinetic energy, $E = \frac{P^2}{2m}$

$\therefore \frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2 = (2)^2 = 4$

$\therefore E_2 = 4E_1$

$\therefore E_2 - E_1 = 3E_1$

5. (B)

Magnification, $m = -\frac{1}{4}$

(Since the image is real and inverted, it is taken as negative)

$\therefore m = \frac{v}{u} = -\frac{1}{4}$

$\therefore v = -\frac{u}{4}$

We have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$-\frac{4}{u} - \frac{1}{u} = \frac{1}{f}$

$-\frac{5}{u} = \frac{1}{f}$

$\therefore u = -5f$

6. (C)

$\alpha = 4\ rad/s^2$

Centripatal (radial) acceleration, $a_r = r\omega^2$

Tangential acceleration, $a_t = r\alpha$

If $a_r = a_t$, then $r\omega^2 = r\alpha$

$\therefore \omega^2 = \alpha = 4$

$\omega = \sqrt{4} = 2\ rad/s$

$\omega = \omega_0 = \alpha t = 0 + \alpha t = \alpha t$

$\therefore 2 = 4t$

or $t = \frac{1}{2} s$

7. (D)

EMF, $e = \pi r^2 B f = \pi r^2 B \frac{\omega}{2\pi} = \frac{1}{2} B r^2 \omega$

$= \frac{1}{2} \times 0.2 \times 10^{-4} \times (1)^2 \times 5$

$= 0.5 \times 10^{-4}\ V = 50\ \mu V$

8. (C)

$E = \sigma A T^4$

$E' = \sigma A' T'^4$

$A = L \times B$

$$A' = \frac{L}{3}\times\frac{B}{3} = \frac{LB}{9} = \frac{A}{9}$$

$$T = 27\ ^\circ C = 300\ K$$

$$T' = 327\ ^\circ C = 600\ K$$

$$\frac{E'}{E} = \frac{A'}{A}\left(\frac{T'}{T}\right)^4 = \frac{1}{9}(2)^4$$

$$\therefore E' = \frac{16E}{9}$$

9. (A)

$R = 0.4$ m, $M = 1$ kg, $\alpha = 10$ rad/s^2

$$I = \frac{MR^2}{2} = \frac{1\times(0.4)^2}{2} = 0.08\ kg\ m^2$$

Torque, $\tau = I\alpha = 0.08 \times 10 = 0.8$ N-m

Also, $\tau = FR$

or $F = \frac{\tau}{R} = \frac{0.8}{0.4} = 2N$

10. (D)

11. (A)

$$\frac{V_2}{V_1} = \frac{1}{8},\ \gamma = \frac{5}{3}$$

For adiabatic process,

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = (8)^{5/3} = (2)^5 = 32$$

12. (A)

13. (C)

Distance between the first minima $= \frac{2\lambda D}{a} = 5$ mm $= 5 \times 10^{-3}$ m

$\lambda = 6000$ Å $= 6 \times 10^{-7}$ m, $D = 80$ cm $= 0.8$ m

$$\therefore \frac{2\times6\times10^{-7}\times0.8}{a} = 5\times10^{-3}$$

$$\therefore a = \frac{2\times6\times10^{-7}\times0.8}{5\times10^{-3}} = 0.192\times10^{-3}\ m$$

$$= 0.192\ mm$$

14. (B)

$d = 1$ m, $A = 10^{-3}$ m^2, $K = 96$ cal/s m °C, $L = 8 \times 10^4$ cal/kg

$t = 1$ min $= 60$ s, $\theta_1 = 0$°C, $\theta_2 = 100$ °C

$$Q = \frac{KA\Delta\theta t}{d} = mL$$

$\therefore\ m = \dfrac{KA\,\Delta\theta\, t}{L} = \dfrac{96\times10^{-3}\times100\times60}{80\times10^{4}}$

$= 7.2\times10^{-3}$ kg

15. (A)

16. (B)

$T = \dfrac{2\pi}{\sqrt{3}}$ s, ZA = 4 cm

$\therefore\ A = 2$ cm $= 2\times10^{-2}$ m

Acceleration, $a = \omega^2 x$

Velocity, $v = \omega\sqrt{A^2 - x^2}$

$\therefore\ \omega^2 x = \omega\sqrt{A^2 - x^2}$

$\omega x = \sqrt{A^2 - x^2}$

$\omega^2 x^2 = A^2 - x^2$

$\therefore\ 3x^2 = A^2 - x^2 \qquad \left(\therefore \omega = \dfrac{2\pi}{T} = \sqrt{3}\right)$

$\therefore\ 4x^2 = A^2$

$c = \dfrac{A}{2} = \dfrac{2}{2} = 1$ cm

17. (D)

In a pipe closed at one end only odd harmonics of the fundamental are produced. If f is the fundamental frequency then the harmonics produced are 3f, 5f, 7f, ……

The difference between the successive overtones is 2f.

$\therefore$ 250 Hz – 150 Hz = 2f

$\therefore$ 100 Hz = 2f

$\therefore$ f = 50 Hz

18. (C)

When two bodies of equal masses collide elastically, they exchange their velocities. Since the two masses are exchanging their velocities, their masses must be equal.

Hence, $\dfrac{m_a}{m_b} = 1$

19. (A)

Half life, T = 15 days, time t = 60 days = 4T

The number of nuclei remaining is given by

$N = N_0\left(\dfrac{1}{2}\right)^n = N_0\left(\dfrac{1}{2}\right)^4 \qquad$ n = no. of half-lives

$= \dfrac{1}{16}N_0 = \dfrac{1}{16}\times8\times10^{16}$

$= 0.5\times10^{16}$

$\therefore$ No. of nuclei decayed $= N_0 - N$

$= 8\times10^{16} - 0.5\times10^{16} = 7.5\times10^{16}$

20. (D)

For maxima, path difference = $n\lambda$

For first maximum, $n = 1$

$\therefore$ Path difference, $\lambda = 6300$ Å

21. (B)

Excess pressure in a water drop = $\frac{2T}{R}$

$$T = 7.2 \times 10^{-2}\ Nm^{-2}$$

$$\therefore\ 72 = \frac{2 \times 7.2 \times 10^{-2}}{R}$$

$$\therefore\ R = \frac{2 \times 7.2 \times 10^{-2}}{72} = 2 \times 10^{-3}\ m = 2\ mm$$

22. (D)

The velocity of the body when it reaches the surface of the lake is given by

$$v^2 = 2gh \quad \ldots(1)$$

When the body is in air the gravitational force acting on it is

$$F = mg = V\rho g \quad \ldots(2)$$

Where V is the volume of the body.

When the body enters the lake, there will be an upthrust acting on the body.

The upthrust is given by

$$U = V\delta g$$

Since $\delta > \rho$, upthrust will be greater then the gravitational force. The net force acting on the body will be

$$F' = V\delta g - V\rho g = Vg = (\delta - \rho) \quad \ldots(3)$$

By eqs. (2) and (3)

$$\frac{F'}{F} = \frac{\delta - \rho}{\rho}$$

$$\frac{a}{g} = \frac{\delta - \rho}{\rho}$$

$$\therefore\ a = \left(\frac{\delta - \rho}{\rho}\right)g \qquad \therefore\ \frac{g}{a} = \frac{\rho}{\delta - \rho}$$

If a is the acceleration retardation in the liquid then

$$v^2 = 2ad \quad \ldots(4)$$

BY eqs (1) and (4)

$$2ad = 2gh$$

$$\therefore\ d = \frac{g}{a}h \qquad \therefore\ d = \frac{\rho}{\delta - \rho}h$$

23. (B)

If d is the initial distance between the plates then capacitance is given by

$$C = \frac{kA \in_0}{d}$$

When a plate of thickness t = 2 mm is inserted between the plates the capacitance,

$$C_1 = \frac{kA\epsilon_0}{\left(d - t + \frac{t}{k}\right)}$$

If distance between the plates is increased by $x = 1.6$ mm

Capacitance becomes, $C_2 = \frac{kA\epsilon_0}{\left(d + x - t + \frac{t}{k}\right)}$ But $C_2 = C$

$$\therefore\ d + x - t + \frac{t}{k} = d$$

$$\therefore\ t - x = \frac{t}{k}$$

$$\therefore\ 2 - 1.6 = \frac{2}{k}$$

$$\therefore\ 0.4 = \frac{2}{k} \qquad \therefore\ k = 5$$

24. (A)

The electric field at the centre due to a small element ds of the ring is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, ds}{r^2}$$

The field can be resolved along X and Y directions as shown in the figure.
Similarly field due to symmetrically situated another element can also be resolved along X and Y directions. The X-components are in the same direction and get added while the Y-components are in opposite directions and cancel each other.
Hence, the total field due to the range is

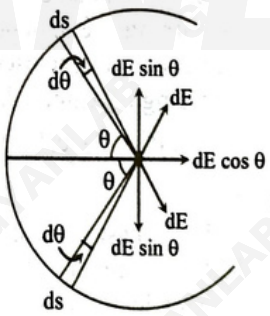
$$E = \int dE\cos\theta$$

$$\therefore\ E = \int \frac{1}{4\pi\epsilon_0}\frac{\lambda\, ds}{r^2}\cos\theta$$

$$= \int \frac{1}{4\pi\epsilon_0}\frac{\lambda\, d\theta\cos\theta}{r^2} \qquad = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}[\sin\theta]_{0\pi/2}^{\pi/2} \qquad = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}[1-(-1)]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}2 \qquad = \frac{\lambda}{2\pi\epsilon_0 r}$$



25. (C)

In the region $0 \le x \le 1$, the potential energy of the particle is E.
Total energy is nE.
Hence, kinetic energy, $K = nE - E = (n-1)E$
Its momentum, $p_1 = \sqrt{2mK} = \sqrt{2m(n-1)E}$

$$\therefore\ \lambda_1 = \frac{h}{p_1} = \frac{h}{\sqrt{2m(n-1)E}}$$

In the region $x > 1$, PE is zero.

Hence, total energy is kinetic energy.

$K = nE$

$\lambda_2 = \frac{h}{p_2} = \frac{h}{\sqrt{2mK}}$

$\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{n}{n-1}}$

26. (B)

$\tan\phi = \tan 45° = \frac{X_L}{R}$

$\therefore \frac{X_L}{R} = 1 \quad \text{or} \quad X_L = R$

$\therefore 2\pi f L = R$

or $L = \frac{R}{2\pi f} = \frac{100}{2\pi \times 1000} = \frac{0.05}{\pi}$ H

27. (C)

$I = MK^2$

For a ring, $MR^2 = MK_r^2 \quad \therefore K_r = R$

For a disc, $\frac{MR^2}{2} = MK_d^2 \quad \therefore K_d = \frac{R}{\sqrt{2}}$

$\therefore \frac{K_r}{K_d} = \sqrt{2}$

28. (C)

Current of 5A and 2A are enclosed in the loop.
The currents are in opposite direction.
The net current enclosed by the loop is 3A.
According to Ampere law,

$\oint \vec{B}\, d\vec{l} = \mu_0 I = 3\mu_0$

29. (A)

At the highest point the particle will come to rest momentarily, hence it is at extreme position and has maximum force and acceleration. Since the spring is unstretched, the restoring force is provided by the weight of the particle

$\therefore mA\omega^2 = mg$

or $A\omega^2 = g$

$\therefore A = \frac{g}{\omega^2}$

$\omega = 2\pi f = 2\pi \times 5 = 10\pi$

$\therefore A = \frac{10}{100\pi^2} = \frac{1}{10\pi^2}$

$V_{max} = A\omega = \frac{1}{10\pi^2} \times 10\pi = \frac{1}{\pi}$

30. (B)

$h\nu = 4.2$ eV, $\omega_0 = 2.4$ eV, $(KE)_{max} = 4.2 - 2.4 = 1.8$ eV
The electrons will not be able to escape from the surface when its potential becomes 1.8 V.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$\therefore\ 1.8 = 9 \times 10^9 \times \frac{q}{0.1}$

$\therefore\ q = 2 \times 10^{-8}$ C

$\therefore\ ne = q$

$n = \frac{q}{e} = \frac{2\times10^{-11}}{1.8\times10^{-14}} = 1.25 \times 10^8$

31. (B)

Time of flight, $T = \frac{2u\sin\theta}{g} = 2$ s

$\therefore\ u\sin\theta = g = 10$ m/s

Maximum height, $H = \frac{u^2\sin^2\theta}{2g} = \frac{(10)^2}{2\times10} = \frac{100}{20} = 5$ m

32. (C)

For an ideal transformer,

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Hence, if $V_s < V_p$ then $I_p < I_s$

33. (A)

n = 264 Hz, V = 300 m/s

For fundamental frequency, $n = \frac{V}{4l}$

$l = \frac{V}{4n} = \frac{330}{4\times264} = 0.3125$ n $= 31.25$ cm

For fundamental mode, $l = \frac{\lambda}{4}$

other possible lengths are $\frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots$

i.e. 93.75 cm, 156.25 cm, ……

Hence, 62.50 m is not possible.

34. (A)

$U = 4\pi R^2 T$

$U' = 512 \times 4\pi r^2 T \quad \left(r = \frac{R}{8}\right)$

$= 512 \times \frac{4\pi R^2 T}{64} = 8\,(4\pi R^2 T) = 8U$

35. (A)

36. (D)

According to Wien's law, λT = constant

$\therefore\ T \propto \frac{1}{\lambda}$

wavelength of blue light is less than that of yellow light.
Hence, temperature of Q is greater than temperature of P.

37. (D)

Focal length in air is given by

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(1)$$

Focal length in liquid is given by

$$\frac{1}{f'} = (n' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(2)$$

where $n' = \frac{n_g}{n_\ell} = \frac{3}{2} \times \frac{5}{9} = \frac{5}{6}$

Dividing eq. (1) and (2),

$$\frac{f'}{f} = \frac{n_g - 1}{n' - 1} = \frac{\frac{3}{2} - 1}{\frac{5}{6} - 1} = \frac{\frac{1}{2}}{-\frac{1}{6}} = -3$$

$\therefore\ f' = -3f = -3 \times 15 = -45$ cn

38. (B)

Emf induced, $e = B_H l V$

$V^2 = 2gh = 2 \times 10 \times 10 = 200$

$\therefore\ V = 10\sqrt{2}$ m/s

$l = 2500$ m, $B_H = 2 \times 10^{-5}$ T

$\therefore\ e = 2 \times 10^{-5} \times 2500 \times 10\sqrt{2} = 0.5\sqrt{2}$

$$I = \frac{e}{R} = \frac{0.5\sqrt{2}}{25\sqrt{2}} = \frac{0.5}{25} = 0.02\ A$$

39. (C)

Energy required to escape the earth's gravitational field is $\frac{1}{2}mV_e^2$

Energy given to the body is $= \frac{1}{2}m(3V_e)^2$

$$= \frac{9}{2}mV_e^2$$

$\therefore$ If V is the velocity of the body when it has escaped from earth's gravitational field then

$$\frac{1}{2}mV^2 = \frac{9}{2}mV_e^2 - \frac{1}{2}mV_e^2$$

$$\therefore\ \frac{1}{2}mV^2 = 4mV_e^2$$

$$\therefore\ V^2 = 8V_e^2$$

$$V = 2\sqrt{2}\,V_e$$

40. (A)

$$g' = g\left(1 - \frac{d}{R}\right)$$

$$\therefore\ \frac{g}{n} = g\left(1 - \frac{d}{R}\right)$$

$\therefore \frac{1}{n} = \left(1 - \frac{d}{R}\right)$

$\therefore \frac{d}{R} = 1 - \frac{1}{n} \quad = \frac{n-1}{n}$

$\therefore d = \frac{R(n-1)}{n}$

41. (D)

$R = 500\ \Omega,\ \phi = 60°$

When capacitance is removed

$\tan\phi = \tan 60° = \frac{X_L}{R}$

$\therefore \frac{X_L}{R} = \sqrt{3}$ OR $X_L = R\sqrt{3}$

When inductance is removed,

$\frac{X_C}{R} = \tan 60° = \sqrt{3}$

$\therefore X_C = R\sqrt{3}$

$\therefore X_C = X_L$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = R = 500\ \Omega$

42. (B)

Gyromagnetic ratio $= \frac{e}{2m}$

It is independent of the orbit of the electron.

43. (C)

$F_1 = K_1 x \qquad F_2 = K_2 x \qquad F = (K_1 + K_2)x$

$\therefore T_1 = 2\pi\sqrt{\frac{m}{K}} \qquad T_2 = 2\pi\sqrt{\frac{m}{K}}$

$\therefore T_1^2 = 4\pi^2 \frac{m}{K_1} \qquad T_2^2 = 4\pi^2 \frac{m}{K_2}$

$T^2 = 4\pi^2 \frac{m}{K_1 + K_2}$

$\therefore \frac{1}{T^2} = \frac{K_1 + K_2}{4\pi^2 m} = \frac{K_1}{4\pi^2 m} + \frac{K_2}{4\pi^2 m}$

$= \frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{T_1^2 + T_2^2}{T_1^2 T_2^2}$

$\therefore T^2 = \frac{T_1^2 T_2^2}{T_1^2 + T_2^2}$

$\therefore T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

44. (A)

$y_1 = a \sin 2000\pi t$

$\therefore 2\pi n_1 = 2000\pi \qquad \therefore n_1 = 1000\ \text{Hz}$

$y_2 = a \sin 2008\pi t$

$\therefore\ 2\pi n_2 = 2008\pi \qquad \therefore\ n_2 = 1004$ Hz

$\therefore$ Beat frequency $= n_2 - n_1$

$= 4$ Hz

45. (D)

If I′ is the intensity of each wave, then resultant intensity is given by

$$I = 4I' \cos^2 \frac{\phi}{2}$$

I will have maximum value when $\cos^2 \frac{\phi}{2} = 1$

$\therefore$ maximum intensity, $I_0 = 4I'$

When path difference is $\frac{\lambda}{4}$, the phase difference, $\phi = \frac{\pi}{2}$

The resultant intensity, $I = 4I' \cos^2 \frac{\pi}{4} = 4I' \times \frac{1}{2} = 2I'$

$$\therefore\ \frac{I}{I_0} = \frac{1}{2}$$

46. (A)

47. (B)

$$s_1 = \frac{I_g G}{3I - I_g},\quad s_2 = \frac{I_g G}{4I - I_g}$$

$$\therefore\ \frac{s_2}{s_1} = \frac{3I - I_g}{4I - I_g}$$

48. (B)

If the length of potentiometer wire increases, the potential gradient will decrease. Hence to balance the same p.d., the length will increase.

49. (B)

$$V = \frac{1}{4\pi \epsilon_0}\frac{q}{R} - \frac{1}{4\pi \epsilon_0}\frac{q}{5R}$$

$$= \frac{1}{4\pi \epsilon_0}\frac{q}{R}\left(1 - \frac{1}{5}\right)$$

$$= \frac{1}{4\pi \epsilon_0}\frac{q}{R}\frac{4}{5}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{25R^2}$$

$$\therefore\ \frac{E}{V} = \frac{1}{20R} \quad \text{or} \quad E = \frac{V}{20R}$$

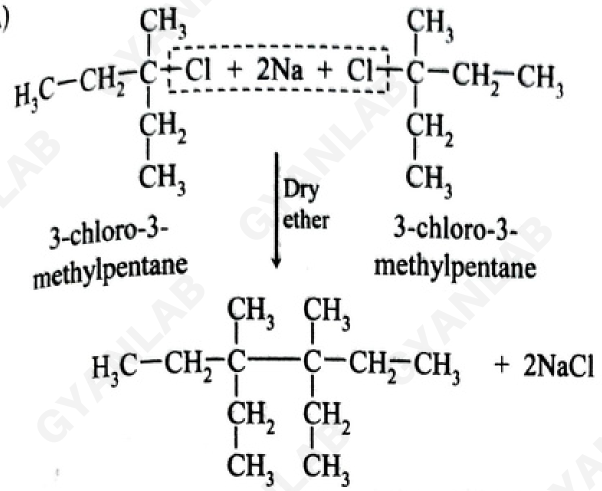
50. (B)

$$M = \frac{enh}{4\pi m_e}$$

CHEMISTRY

51. (A)
Lanthanoids react with nitrogen and halogens to give nitrides and halides of the formulae LnN and LnX_3 respectively.

52. (A)



3,4-Diethyl-3,4-dimethylhexane

53. (B)

$$2NO + Cl_2 \rightarrow 2NOCl$$

Rate of reaction $= -\frac{1}{2}\frac{d[NO]}{dt} = -\frac{d[Cl_2]}{dt} = \frac{1}{2}\frac{d[NOCl]}{dt}$

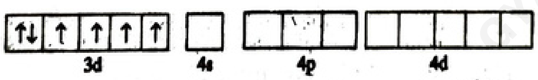
$$\therefore \frac{d[NO]}{dt} = \frac{d[NOCl]}{dt}$$

54. (B)

Formation of $[CoF_6]^{3-}$

It is an example of sp^3d^2 hybridization. An octahedral complex shows paramagnetic behaviour. It utilizes outer 4d orbital in sp^3d^2 hybridization. It is therefore called **outer orbital** or **high spin** or **spin free complex**.

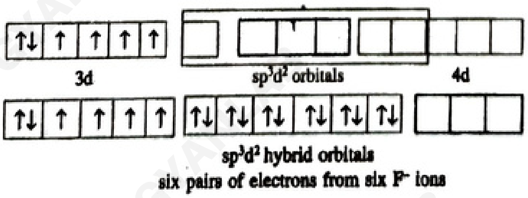
Orbitals of Co^{3+} ion



Since F^- is a weak ligand, there is no spin pairing effect and Co^{3+} possesses 4 unpaired electrons.

Co^{3+} undergoing sp^3d^2 hybridization

$[CoF_6]^{3-}$
(Outer orbital or High spin complex



55. (C)

56. (C)

$$\underset{\text{Glucose}}{\begin{matrix} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{matrix}} \xrightarrow[HNO_3]{(O)} \underset{\text{Saccharic acid}}{\begin{matrix} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \end{matrix}} \xleftarrow[HNO_3]{(O)} \underset{\text{Gluconic acid}}{\begin{matrix} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{matrix}}$$

This confirms the presence of one primary alcoholic group ($-CH_2OH$) in glucose.

57. (A)

Alkyl groups causes +I effect.

58. (C)

59. (D)

$S = 7.2 \times 10^{-7}$ mol dm^{-3}, $K_{sp} = ?$

For AgCl, $K_{sp} = S^2$

$= (7.2 \times 10^{-7})^2 \quad = 5.18 \times 10^{-13}$

60. (D)

$$\underset{\text{Benzoic acid}}{C_6H_5{-}COOH} + PCl_5 \xrightarrow{\Delta} \underset{\text{Benzoyl Chloride}}{C_6H_5COCl} + POCl_3 + HCl$$

61. (D)

$[A]_0 = 20$ m mol, $[A]_t = 10$ m mol

$t = 1.151$ min, $k = ?$

For first order reaction,

$$k = \frac{2.303}{t}\log_{10}\frac{[A]_0}{[A]_t}$$

$$\therefore\ k = \frac{2.303}{1.151\,\text{min}}\log_{10}\left(\frac{20}{10}\right) = \frac{2.303 \times \log 2}{1.151} = \frac{2.303 \times 0.3010}{1.151}$$

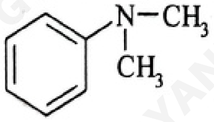
$= 0.60\ \text{min}^{-1}$

62. (B)

(i) $H_3C{-}\underset{\underset{C_2H_7}{|}}{N}{-}C_3H_7$ – Ethyl methyl propyl amine (3° amine)

(ii) $H_2N{-}(CH_2)_6{-}NH_2$ – Hexamethylene diamine (1° amine)

(iii) $C_6H_5{-}NH{-}C_6H_5$ – Diphenyl amine (2° amine)

(iv) 

– N,N-Dimethyl aniline (3° amine)

63. (C)

Except promethium (Pm), all are non-radioactive in nature.

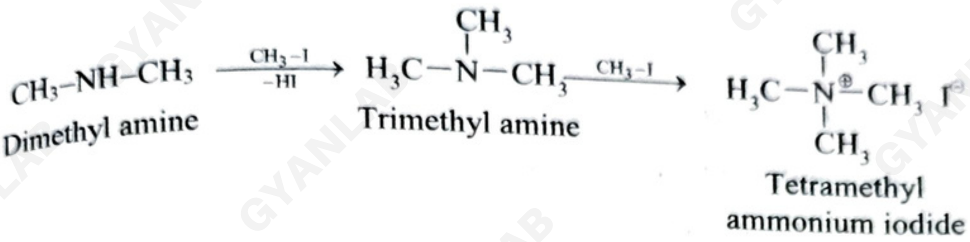
64. (A)

Total no. of voids $= 3 \times 6.022 \times 10^{23}$

$= 1.806 \times 10^{24}$

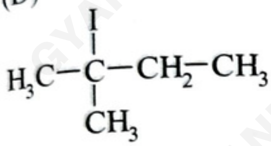
65. (B)
(A) $Na_3[AlF_6]$ – Anionic
(B) $[Co(NO_2)_3(NH_3)_3]$ – Neutral
(C) $[Cu(NH_3)_4]^{2+}$ – Cationic
(D) $[Fe(CN)_6]^{4-}$ – Anionic

66. (C)



67. (A)

68. (D)



2-Iodo-2-methylbutane
(No chiral carbon atom)

69. (D)

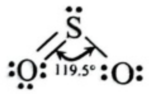
$$2H_2O_2 + ClO_4^{\ominus} \longrightarrow 2O_2 + ClO_2^{\ominus} + 2H_2O$$

70. (B)

$$\text{Raffinose} \xrightarrow{\text{Hydrolysis}} \text{Glucose} + \text{Fructose} + \text{Galactose}$$

71. (C)

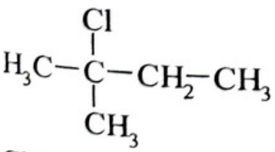
72. (D)



Structure of SO_2

73. (D)
Due to highest bond order in N_2, it has highest bond enthalpy.

74. (A)



2-Chloro-2-methylbutane (No chiral carbon atom)

75. (B)

Radioactive processes follow the first order kinetics.

76. (D)

77. (C)

The cell constant is determined using the 1 M, 0.1 M or 0.01 M KCl solutions.

78. (D)

$$H_{2(g)} + Cl_{2(g)} \rightarrow 2HCl_{(g)}\ ;\ \Delta H = -194\ kJ$$

Heat of formation for 2 moles of HCl $= -194$ kJ

$\therefore$ Heat of formation for 1 mole of HCl $= -97$ kJ

79. (C)

Bohr's frequency rule,

$$\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

80. (B)

$$K_a = 5 \times 10^{-8},\ \alpha = 0.5\% = \frac{0.5}{100} = 5 \times 10^{-3}$$

$$K_a = \alpha^2 c$$

$$\therefore\ c = \frac{K_a}{\alpha^2} = \frac{5\times10^{-8}}{(5\times10^{-3})^2} = \frac{5\times10^{-8}}{25\times10^{-6}}$$

$$= 0.002\ M$$

81. (C)

82. (A)

$$c = 0.05\ M,\ k = 0.0118\ S\ cm^{-1}$$

$$\wedge = \frac{1000k}{c}$$

$$= \frac{1000\,cm^3\,L^{-1} \times 0.0118\ S cm^{-1}}{0.05\ mol L^{-1}}$$

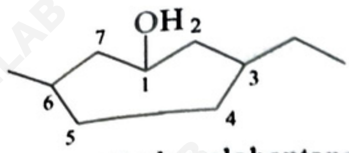
$$= 236\ S\ cm^2\ mol^{-1}$$

83. (A)

As the molecular mass increases, boiling point of aldehydes increases.

$\therefore$ Hexanal has the highest boiling point.

84. (B)



3-Ethyl-6-methyl cycloheptanol

85. (B)

$\rho = 10\ g\ cm^{-3}$, a = 200 pm $= 2 \times 10^{-8}$ cm

For BCC structure, n = 2

$$M = \frac{\rho a^3 N_A}{n}$$

$$= \frac{10 \times (2 \times 10^{-8})^3 \times 6.022 \times 10^{23}}{2} = \frac{10 \times 8 \times 10^{-24} \times 6.022 \times 10^{23}}{2}$$

$$= 24.1\ g\ mol^{-1}$$

86. (C)

$$C_2H_{6(g)} + 3.5O_{2\,(g)} \rightarrow 2CO_{2\,(g)} + 3H_2O_{(l)}$$

$$\Delta n_{(g)} = 2 - 4.5 = -2.5$$

$$\Delta H - \Delta U = \Delta n_{(g)}RT$$

$$= -2.5\ mol \times 8.314\ J\ K^{-1}\ mol^{-1} \times 298\ K$$

$$= -6193.93\ J \quad = -6.193\ kJ \quad \approx -6.2\ kJ$$

87. (A)

$W_2 = 5$ g, $\quad W_1 = 100$ g

$M_2 = 180$ g, $\quad \Delta T_f = 2.15$ K

$K_f = ?$

$$\Delta T_f = K_f \frac{1000\ W_2}{M_2\ W_1}$$

$$\therefore\ K_f = \frac{\Delta T_f\ M_2\ W_1}{1000\ W_2} = \frac{2.15 \times 180 \times 100}{1000 \times 5}$$

$$= 7.74\ K\ kg\ mol^{-1}$$

88. (C)

$CH_3{-}CH{=}CH{-}CH_3$ (But-2-ene) and $CH_2{=}CH{-}CH_2{-}CH_3$ (But-1-ene)

But-1-ene and but-2-ene have same molecular formula (C_4H_8) and the same carbon skeleton but the double bonds are located at different positions.

89. (A)

90. (C)

$$H_3C{-}\underset{\displaystyle CH_3}{\underset{|}{CH}}{-}CH_2{-}OH$$

(Isobutyl alcohol)

Carbinol system – Isopropyl carbinol

91. (B)

$F-CH_2-COOH > Cl-CH_2-COOH > Br-CH_2-COOH > I-CH_2-COOH$

Higher the electronegativity of halogen, greater is the stabilization of the conjugate base, stronger is the acid.

92. (C)

$CH_3-CH_2-O-CH_3$ (Methoxyethane) and $CH_3-CH_2-CH_2-OH$ (Propan-1-ol)

Both have same molecular formula (C_3H_8O) but different functional groups.

93. (C)

94. (C)

$K_H = 0.16 \text{ mol L}^{-1} \text{ bar}^{-1}$, $S = 0.08 \text{ mol L}^{-1}$

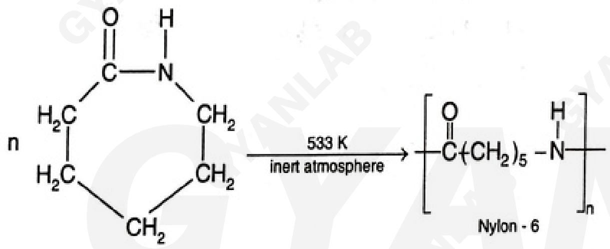
According to Henry's law,

$$S = K_H P$$

$$\therefore P = \frac{S}{K_H} = \frac{0.08 \text{ mol L}^{-1}}{0.16 \text{ mol L}^{-1} \text{ bar}^{-1}} = 0.5 \text{ bar}$$

95. (A)

96. (C)



97. (D)

$V_1 = 11.2 \text{ dm}^3$, $P_1 = 105 \text{ kPa}$

$V_2 = ?$, $P_2 = 420 \text{ kPa}$

According to Boyle's law,

$$P_1V_1 = P_2V_2$$

$$\therefore V_2 = \frac{P_1 V_1}{P_2} = \frac{105 \text{ kPa} \times 11.2 \text{ dm}^3}{420 \text{ kPa}} = 2.8 \text{ dm}^3$$

98. (A)

Internal energy is a state function.

99. (C)

$$Ca^{2+}_{(aq)} + 2e^- \longrightarrow Ca_{(s)}$$

40 g of Ca requires 2F of electricity

$$\therefore 10 \text{ g of Ca} = \frac{2 \times 10}{40} = 0.5 \text{ F of electricity}$$

100. (D)

$$CH_3-CH=CH-CH_2-CHO \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } LiAlH_4} CH_3-CH=CH-CH_2-CH_2-OH$$

$LiAlH_4$ does not reduce the isolated olefinic bond and hence it can reduce unsaturated aldehydes and ketones to unsaturated alcohols.

Section II

MATHEMATICS

101.(A)

$\sin^{-1}[\sin(-600)^\circ] + \cot^{-1}\left(-\sqrt{3}\right)$

$= \sin^{-1}\left[-\sin\left(360^\circ+180^\circ+60^\circ\right)\right]+\left[-\cot^{-1}\left(\sqrt{3}\right)\right]$

$= \sin^{-1}\left[-\sin\left(3\pi+60^\circ\right)\right]+\left[-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

$= \sin^{-1}\left[\sin 60^\circ\right]-\frac{\pi}{6} = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6}$

102.(C)

$y = \frac{dp}{dx}+\sqrt{a^2p^2-b^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)+\sqrt{a^2\left(\frac{dy}{dx}\right)^2-b^2}$

$\therefore\ y-\frac{d^2y}{dx^2} = \sqrt{a^2\left(\frac{dy}{dx}\right)^2-b^2}$

Squaring both sides, we write

$y^2+\left(\frac{d^2y}{dx}\right)^2-2y\frac{d^2y}{dx^2} = a^2\left(\frac{dy}{dx}\right)^2-b^2$

$\therefore$ order $= 2$ and degree $= 2$

103.(D)

We have $\frac{dA}{dt} = 2$ and $A = 4\pi r^2$

$\therefore\ \frac{dA}{dt} = 8\pi r\frac{dr}{dt} \Rightarrow 2 = 8\pi(6)\frac{dr}{dt}$

$\therefore\ \frac{dr}{dt} = \frac{1}{24\pi}$

Here $V = \frac{4}{3}\pi r^3$

$\therefore\ \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$= 4\pi(6)^2\left(\frac{1}{24\pi}\right) = 6\ cm^3/sec$

104.(D)

$P(X=x) = \frac{1}{32}\,{}^5C_x$, where $x = 0, 1, 2, 3, 4, 5$

$= 0$, otherwise

$\therefore\ P(X \le 2) = P(X=0)+P(X=1)+P(X=2)$

$= \frac{1}{32}\left[{}^5C_0+{}^5C_1+{}^5C_2\right] = \frac{1}{32}(1+5+10) = \frac{16}{32} = \frac{1}{2}$

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$
$$= \frac{1}{32}\left[{}^5C_3 + {}^5C_4 + {}^5C_5\right] = \frac{1}{32}[10 + 5 + 1] = \frac{16}{32} = \frac{1}{2}$$

105.(D)

We have $\bar{a}\cdot(\bar{b}+\bar{c}) = 0, \bar{b}\cdot(\bar{c}+\bar{a}) = 0$ and $\bar{c}\cdot(\bar{a}+\bar{b}) = 0$

$\therefore\ \bar{a}\cdot\bar{b}+\bar{a}\cdot\bar{c} = 0$ ...(1)

$\bar{b}\cdot\bar{c}+\bar{b}\cdot\bar{a} = 0$ ...(2)

$\bar{c}\cdot\bar{a}+\bar{c}\cdot\bar{b} = 0$ ...(3)

From (1), (2) and (3), we get

$$2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}) = 0$$

Now $|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})$

$\therefore\ |\bar{a}+\bar{b}+\bar{c}|^2 = (5)^2+(4)^2+(3)^2+2(0) = 50$

106.(C)

We have $\frac{n!}{2!(n-2)!}\times\frac{4!(n-4)!}{n!} = \frac{2}{1}$

$\therefore\ \frac{(4\times3)}{(n-2)(n-3)} = 2 \quad\Rightarrow n^2-5n+6 = 6$

$\therefore\ n(n-5) = 0 \quad\Rightarrow n = 5$

107.(C)

From the data given, symbolic form of the given statement is

$(\sim p \vee \sim q) \to (r \wedge s)$

$\equiv \sim(p \wedge q) \to (r \wedge s)$

108.(B)

$$a = \lim_{n\to\infty}\frac{1+2+3+...+n}{n^2}$$
$$= \lim_{n\to\infty}\frac{n(n+1)}{2(n)(n)} = \lim_{n\to\infty}\left(\frac{1}{2}\right)\left(\frac{n+1}{n}\right) = \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = \frac{1}{2}$$

$$b = \lim_{n\to\infty}\frac{1^2+2^2+3^2+...+n^2}{n^3}$$
$$= \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3} = \lim_{n\to\infty}\frac{2n^2+3n+1}{6n^2}$$
$$= \lim_{n\to\infty}\left(\frac{1}{6}\right)\left(2+\frac{3}{n}+\frac{1}{n^2}\right) = \frac{2}{6} = \frac{1}{3}$$

Thus $a = \frac{1}{2}$ and $b = \frac{1}{3} \Rightarrow 2a = 1 = 3b = 1$

109.(B)

$(1+\omega)^7 = A + B\omega$

$\therefore\ A + B\omega = (-\omega^2)^7$ ...$[\because 1+\omega+\omega^2 = 0]$

$= (-1)\,\omega^{14} = -\omega^{12}\,\omega^2 = -\omega^2 = (1+\omega)$

$\therefore\ A = 1, B = 1$

110.(D)

We have $\bar{a}\cdot(\bar{b}\times\bar{c})=4$

Volume of required parallelopiped is

$(\bar{a}+2\bar{b})\cdot\left[(\bar{b}+2\bar{c})\times(\bar{c}+2\bar{a})\right]$

$=(\bar{a}+2\bar{b})\cdot\left[(\bar{b}\times\bar{c})+2(\bar{c}\times\bar{c})+2(\bar{b}\times\bar{a})+4(\bar{c}\times\bar{a})\right]$

$=\bar{a}\cdot(\bar{b}\times\bar{c})+\bar{a}(0)+2\bar{a}\cdot(\bar{b}\times\bar{a})+4\bar{a}\cdot(\bar{c}\times\bar{a})+2\bar{b}\cdot(\bar{b}\times\bar{c})+4\bar{b}(0)+4\bar{b}\cdot(\bar{b}\times\bar{a})+8\bar{b}\cdot(\bar{c}\times\bar{a})$

$=\bar{a}\cdot(\bar{b}\times\bar{c})+0+0+0+0+0+0+8\bar{a}\cdot(\bar{b}\times\bar{c}) \quad =9\left[\bar{a}\cdot(\bar{b}\times\bar{c})\right] \quad =9(4)=36$

111.(D)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{9}{16}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$
Total		$\frac{1}{2}$	$\frac{5}{8}$

Variance $=\sum p_i x_i^2-\left(\sum p_i x_i\right)^2$

$=\frac{5}{8}-\left(\frac{1}{2}\right)^2=\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$

112.(C)

$\cos x \sin y\, dx+\sin x \cos y\, dy=0$

$\therefore \cos x \sin y\, dx=-\sin x \cos y\, dy$

$\therefore \int\frac{\cos x}{\sin x}dx=-\int\frac{\cos y}{\sin y}dy$

$\therefore \log|\sin x|=-\log|\sin y|+c_1$

$\therefore \log|\sin x|+\log|\sin y|=c_1 \Rightarrow \log[\sin x \sin y]=c_1$

$\therefore \sin x \sin y=e^{c_1}=c$

113.(C)

$\bar{a}+\bar{b}+\bar{c}=0 \Rightarrow \bar{c}=-(\bar{a}+\bar{b})$ and let angle between $\bar{a}$ and $\bar{b}$ be θ

$\therefore |\bar{c}|^2=(\bar{a}+\bar{b})^2=|\bar{a}|^2+|\bar{b}|^2+2\bar{a}\cdot\bar{b}$

$=|\bar{a}|^2+|\bar{b}|^2+2|\bar{a}|\cdot|\bar{b}|\cdot\cos\theta$

$\therefore (7)^2=(3)^2+(5)^2+2(3)(5)\cos\theta$

$\therefore 49=9+25+30\cos\theta \Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

114.(B)

Let $I = \int \sec^4 x \tan^4 x\, dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\therefore I = \int \sec^2 x (\sec^2 x)(\tan^4 x) dx$

$$= \int (1+t^2)(t)^4 dt = \int (t^4 + t^6) dt = \frac{t^5}{5} + \frac{t^7}{7} + c = \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + c$$

115.(B)

Equation of line through $\left(\frac{-1}{2}, 1\right)$ and (1, 2) is

$$\frac{y-2}{1-2} = \frac{x-1}{\frac{-1}{2}-1} \Rightarrow \frac{y-2}{-1} = \frac{x-1}{\left(\frac{-3}{2}\right)}$$

$\therefore y - 2 = \frac{2(x-1)}{3} \Rightarrow 3y - 6 = 2x - 2$

$\therefore 2x - 3y = -4 \Rightarrow \frac{2x}{-4} - \frac{3y}{-4} = 1 \Rightarrow \frac{x}{-2} + \frac{y}{\left(\frac{4}{3}\right)} = 1$

116.(B)

The required Cartesian equation of line is

$$\frac{x-2}{1-2} = \frac{y-2}{3-2} = \frac{z-1}{0-1} \text{ i.e. } \frac{x-2}{-1} = \frac{y-2}{1} = \frac{z-1}{-1}$$

117.(A)

Mean of given numbers is $\frac{2+3+11+x}{4}$ i.e. $\frac{16+x}{4}$.

Variance $= \frac{1}{n}\Sigma(x_i - \overline{x})^2$

$$\therefore \frac{49}{4} = \frac{1}{4}\left[\left(\frac{16+x}{4}-2\right)^2 + \left(\frac{16+x}{4}-3\right)^2 + \left(\frac{16+x}{4}-11\right)^2 + \left(\frac{16+x}{4}-x\right)^2\right]$$

$$\therefore 49 = \frac{(8+x)^2}{16} + \frac{(4+x)^2}{16} + \frac{(x-28)^2}{16} + \frac{(16-3x)^2}{16}$$

$\therefore (49)(16) = (64 + x^2 + 16x) + (16 + x^2 + 8x) + (x^2 - 56x + 784) + (256 + 9x^2 - 96x)$

$\therefore 784 = 12x^2 - 128x + 80 + 784 + 256$

$\therefore 12x^2 - 128x + 336 = 0 \Rightarrow 3x^2 - 32x + 84 = 0$

$$\therefore x = \frac{32 \pm \sqrt{(32)^2 - (4)(3)(84)}}{2(3)} = \frac{32 \pm \sqrt{1024 - 1008}}{6} = \frac{32 \pm 4}{6}$$

$\therefore x = \frac{36}{6}$ or $x = \frac{28}{6} \Rightarrow x = 6, \frac{14}{3}$

118.(A)

For the given ellipse, we have a = 2b

$\therefore \frac{x^2}{4b^2} + \frac{y^2}{b^2} = 1 \Rightarrow x^2 + 4y^2 = 4b^2$

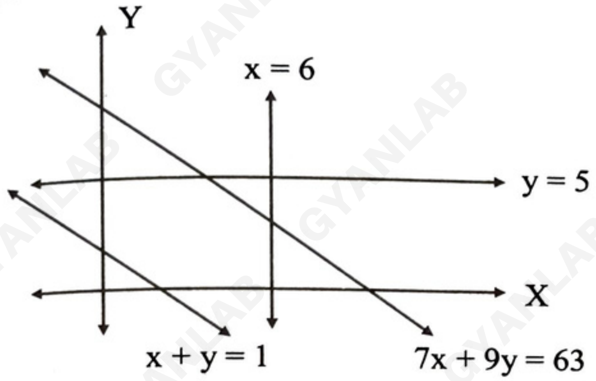
Differentiating both sides w.r.t. x, we get

$$2x + 8y\frac{dy}{dx} = 0 \quad \Rightarrow \quad x + 4y\frac{dy}{dx} = 0$$

119.(A)

Lines are x + y = 1, 7x + 9y = 63, y = 5, x = 6

The required area is shaded.



120.(B)

$$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left[\frac{\left(\frac{3}{4}\right)+\left(\frac{5}{12}\right)}{1-\left(\frac{3}{4}\right)\left(\frac{5}{12}\right)}\right]$$

$$= \tan^{-1}\left(\frac{36+20}{48-15}\right) = \tan^{-1}\left(\frac{56}{33}\right) = \cos^{-1}\left(\frac{33}{65}\right)$$

121.(A)

Required cofactors are as follows :

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} = -(12+1) = -13$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} = (4+2) = 6$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -(1-6) = 5$$

122.(D)

$\bar{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$

$\bar{a} = \hat{i} - \hat{j}$, $\bar{A} = \hat{i} + \hat{j} + \hat{k}$, $\bar{B} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\bar{n}$ is $\perp$er to $\bar{A}$ and $\bar{B}$

$$\therefore \quad \bar{n} = \bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= \hat{i}(3+2)-\hat{j}(3-1)+\hat{k}(-2-1) \quad = 5\hat{i}-2\hat{j}-3\hat{k}$$

$$\bar{a}\cdot\bar{n} \quad = d \quad = \left(\hat{i}-\hat{j}\right)\cdot\left(5\hat{i}-2\hat{j}-3\hat{k}\right) \quad = 5+2=7$$

$\therefore \quad \bar{r}\cdot\left(5\hat{i}-2\hat{j}-3\hat{k}\right)=7 \;\Rightarrow$ Cartesian equation is $5x-2y-3z-7=0$

123.(B)

$$\sqrt{3}\sec x+2 \quad =0$$

$$\therefore \quad \sec x \quad = \frac{-2}{\sqrt{3}} \Rightarrow \cos x = \frac{-\sqrt{3}}{2}$$

$$\therefore \quad \cos x \quad = \cos\left(\pi-\frac{\pi}{6}\right) = \cos\left(\pi+\frac{\pi}{6}\right) \Rightarrow x \quad = \frac{5\pi}{6},\frac{7\pi}{6}$$

124.(A)

$$\text{Let } I \quad = \int \operatorname{cosec}(x-a)\operatorname{cosec} x\,dx$$

$$= \int \frac{dx}{\sin(x-a)\sin x} \qquad\qquad = \int \frac{\sin a}{\sin a\sin(x-a)\sin x}dx$$

$$= \frac{1}{\sin a}\int \frac{\sin(a+x-x)}{\sin(x-a)\sin x}dx \qquad\qquad = \frac{1}{\sin a}\int \frac{\sin-[x-a-x]}{\sin(x-a)\sin x}dx$$

$$= \frac{1}{\sin a}\int \frac{\sin-[(x-a)-(x)]}{\sin(x-a)\sin x}dx \qquad\qquad = \frac{-1}{\sin a}\int \frac{\sin[(x-a)-x]}{\sin(x-a)\sin x}dx$$

$$= \frac{-1}{\sin a}\int \frac{\sin(x-a)\cos x-\cos(x-a)\sin x}{\sin(x-a)\sin x}dx$$

$$= \frac{-1}{\sin a}\int[\cot x-\cot(x-a)]dx \qquad\qquad = \frac{-1}{\sin a}\left[\int \cot x\,dx-\int \cot(x-a)dx\right]$$

$$= \frac{-1}{\sin a}\left[\log|\sin x|-\log|\sin(x-a)|\,\right]+c$$

$$= \frac{1}{\sin a}\left[\log|\sin(x-a)|-\log|\sin x|\right]+c$$

$$= (\operatorname{cosec} a)\left[\log\left|\frac{\sin(x-a)}{\sin x}\right|\right]+c$$

$$= \operatorname{cosec} a\cdot\log|\sin(x-a)\cdot\operatorname{cosec} x|+c$$

125.(B)

$f(x)=2x^3-15x^2-144x-7$

$f'(x)=6x^2-30x-144$

When $f'(x)<0$, we get $x^2-5x-24<0$

$\therefore \; (x-8)(x+3)<0 \Rightarrow -3<x<8$

126.(B)

The equation of the required plane is $(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$ i.e.

$(1 + 2\lambda)x + (2 + \lambda)y + (3 - \lambda)z + (-4 + 5\lambda) = 0$ ...(1)

Since (1) is perpendicular to the plane $5x + 3y - 6z + 8 = 0$, we write

$(1 + 2\lambda)(5) + (2 + \lambda)(3) + (3 - \lambda)(-6) = 0$

$\therefore\ 5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$

$19\lambda = 7 \Rightarrow \lambda = \frac{7}{19}$

Substituting value of λ in eq. (1), we get

$$\left(1+\frac{14}{19}\right)x+\left(2+\frac{7}{19}\right)y+\left(3-\frac{7}{19}\right)z+\left(-4+\frac{35}{19}\right)=0$$

$\therefore\ 33x + 45y + 50z - 41 = 0$

127.(A)

We have $np + npq = 1.8$, where $n = 5$

$\therefore\ np(1 + q) = 1.8 \quad \Rightarrow 5p[1 + (1 - p)] = 1.8$

$\therefore\ 5p(2 - p) = 1.8 \quad \Rightarrow 10p - 5p^2 = 1.8$ i.e.

$5p^2 - 10p + 1.8 = 0 \quad \Rightarrow 5p^2 - 9p - p + 1.8 = 0$

$\therefore\ 5p(p - 1.8) - 1(p - 1.8) = 0 \Rightarrow (5p - 1)(p - 1.8) = 0$

$\therefore\ p = \frac{1}{5}, 1.8$ but $p \le 1 \Rightarrow p = \frac{1}{5} = 0.2$

128.(C)

$y = \sqrt{e^{\sqrt{x}}} \Rightarrow y^2 = e^{\sqrt{x}}$

Takig log on both sides,

$2 \log y = \sqrt{x} \log e \Rightarrow 2 \log y = \sqrt{x}$

Differentiating both sides w.r.t. x, we get

$$\frac{2}{y}\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow \frac{dy}{dx} = y\left[\frac{1}{4\sqrt{x}}\right] = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$$

129.(D)

We have $|\bar{a}\times\bar{b}| = 20$ and we have to find value of $|(3\bar{a}+\bar{b})\times(2\bar{a}+3\bar{b})|$

$(3\bar{a}+\bar{b})\times(2\bar{a}+3\bar{b})$

$= 6(\bar{a}\times\bar{a})+9(\bar{a}\times\bar{b})+2(\bar{b}\times\bar{a})+3(\bar{b}\times\bar{b})$

$= 0+9(\bar{a}\times\bar{b})-2(\bar{a}\times\bar{b})+0 = 7(\bar{a}\times\bar{b})$

Hence area of the required parallelogram $= 7 \times 20 = 140$

130.(C)

$$f(\pi) = \lim_{x\to\pi} f(x) = \lim_{x\to\pi}\frac{4^{x-\pi}+4^{\pi-4}-2}{(x-\pi)^2}$$

Put $x = \pi + h$. As $x \to \pi$, $h \to 0$

$$\therefore\ f(\pi) = \lim_{h\to0}\frac{4^h+4^{-h}-1-1}{h^2} = \lim_{h\to0}\frac{(4^h-1)}{h}\times\lim_{h\to0}\frac{4^{-h}-1}{-h}\times(-1)$$

$= (\log 4)(-\log 4) = (2\log 2)(-2\log 2) = -4(\log 2)^2$

131.(D)

Circle $x^2 + y^2 = (8)^2$, has radius 8 and centre (0, 0). Point $P\left(\frac{2\pi}{3}\right)$ on the circle has coordinates

$P = \left(8\cos\frac{2\pi}{3}, 8\sin\frac{2\pi}{3}\right)$ i.e. $P \equiv (-4,\ 4\sqrt{3})$

Differentiating equation of circle w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \frac{dy}{dx} = \frac{-x}{y} \quad \Rightarrow \left(\frac{dy}{dx}\right)_P = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Hence required equation of tangent is

$$\left(y - 4\sqrt{3}\right) = \frac{1}{\sqrt{3}}(x+4) \Rightarrow x - \sqrt{3}\,y + 16 = 0$$

132.(A)

$2f(x) - 3f\left(\frac{1}{x}\right) = x$ ...(1) and replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x}\right) - 3f(x) = \frac{1}{x}$ ...(2)

$[2\times \text{equation (1)}] + [3\times \text{equation (2)}]$ gives,

$$4f(x) - 9f(x) = 2x + \frac{3}{x} \quad \Rightarrow \quad -5f(x) = 2x + \frac{3}{x}$$

$$\therefore \ f(x) = \frac{-2}{5}x - \frac{3}{5x}$$

$$\therefore \int_1^e f(x)\,dx = \int_1^e\left(\frac{-2}{5}x - \frac{3}{5x}\right)dx = \frac{-2}{5}\int_1^e x\,dx - \frac{3}{5}\int_1^e \frac{1}{x}\,dx$$

$$= \frac{-2}{5}\left[\frac{x^2}{2}\right]_1^e - \frac{3}{5}[\log x]_1^e = \frac{-1}{5}\left(e^2 - 1\right) - \frac{3}{5}(\log e - \log 1) = \frac{-1}{5}e^2 + \frac{1}{5} - \frac{3}{5}$$

$$= -\left(\frac{2+e^2}{5}\right)$$

133.(A)

Volume of cylinder $= \pi r^2 h$

$$\therefore \ \frac{dv}{dt} = \pi r^2\frac{dh}{dt}$$

$$\therefore \ 36 = \pi(3)^2\frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = \frac{4}{\pi}\,\text{m/min}$$

134.(B)

We have $x^2 - 3xy - 4y^2 = 0$ and comparing it with standard equation, we write

$A = 1, H = \frac{-3}{2}, B = -4.$

Equation of bisector of angle of this line is

$$\frac{x^2 - y^2}{A - B} = \frac{xy}{H} \Rightarrow \frac{x^2 - y^2}{1+4} = \frac{xy}{\left(\frac{-3}{2}\right)}$$

$\therefore \ -3x^2 + 3y^2 = 10\,xy \quad \Rightarrow 3x^2 + 10xy - 3y^2 = 0$

Comparing with given equation, we get $k = -10$

135.(A)

$$\left(x\frac{dy}{dx}-y\right)\sin\frac{y}{x}=x^3e^x \quad \ldots(i)$$

Put $\frac{y}{x}=t \Rightarrow \frac{x\frac{dy}{dx}-y}{x^2}=\frac{dt}{dx}$

Hence eq. (i) becomes

$$x^2\left(\frac{dt}{dx}\right)\sin t=x^3e^x \Rightarrow \left(\frac{dt}{dx}\right)\sin t=xe^x$$

$$\therefore \int \sin t\ dt=\int xe^x\,dx$$

$$\therefore -\cos t=xe^x-\int e^x\,dx=xe^x-e^x+c$$

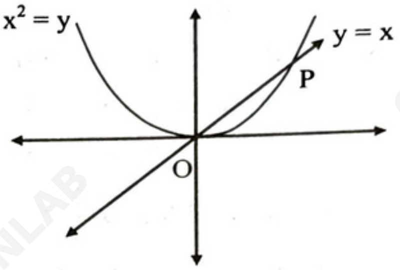
$$\therefore -\cos\left(\frac{y}{x}\right)=e^x(x-1)+c$$

136.(D)

The required area is shaded.

The point of intersection of the curves are

$x^2=x \Rightarrow x(x-1)=0$ i.e. O(0, 0) and P(1, 1)



$$\therefore A=\int_0^1 (x-x^2)\,dx$$

$$=\left[\frac{x^2}{2}\right]_0^1-\left[\frac{x^3}{3}\right]_0^1=\frac{1}{2}-\frac{1}{3}=\frac{1}{6} \text{ sq. units}$$

137.(C)

$$\sin^{-1}\left[\cos\sqrt{\frac{1+x}{2}}\right]+x^x$$

$$=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}\right)\right]+x^x$$

$$=\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}+x^x$$

$$\therefore \frac{dy}{dx}=0-\frac{1}{\sqrt{2}}\cdot\frac{d}{dx}(\sqrt{1+x})+\frac{d}{dx}(x^x)$$

Let $u=x^x \Rightarrow \log u=x\log x$

$$\therefore \frac{1}{u}\frac{du}{dx}=\frac{x}{x}+\log x \quad \Rightarrow \quad \frac{dy}{dx}=x^x(1+\log x)$$

$$\therefore \frac{dy}{dx}=\frac{-1}{\sqrt{2}}\left[\frac{1}{2\sqrt{1+x}}\right]+x^x(1+\log x)$$

$$\therefore \left[\frac{dy}{dx}\right]_{x=1}=\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}}\right)+1=\frac{-1}{4}+1=\frac{3}{4}$$

138.(D)

$\sim[(p\wedge q)\to(\sim p\vee r)]$

$\equiv\sim[\sim(p\wedge q)\vee(\sim p\vee r)]$

$\equiv(p\wedge q)\wedge\sim(\sim p\vee r)$

$\equiv(p\wedge q)\wedge(p\wedge\sim r) \quad \equiv p\wedge q\wedge(\sim r)$

139.(A)

Let $A^{-1}A = I$

$$\therefore \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 5 & 5 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to 3R_2 - R_1$ and $R_3 \to 3R_3 - 2R_1$

$$\begin{bmatrix} 3 & 2 & 6 \\ 0 & 1 & 0 \\ 0 & 11 & 3 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 - 11\,R_2$

$$\begin{bmatrix} 3 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A = \begin{bmatrix} 3 & -6 & 0 \\ -1 & 3 & 0 \\ 9 & -33 & 3 \end{bmatrix}$$

$R_1 \to R_1 - 2R_3$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A = \begin{bmatrix} -15 & 60 & -6 \\ -1 & 3 & 0 \\ 9 & -33 & 3 \end{bmatrix}$$

$R_1 \to \frac{1}{3}R_1$ and $R_3 \to \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} -5 & 20 & -2 \\ -1 & 3 & 0 \\ 3 & -11 & 1 \end{bmatrix}$$

140.(B)

Let Δ OAB be the required triangle. Since ΔOAB is an equilateral triangle.

Slope of line OA = tan 60° = $\sqrt{3}$ and Slope of line OB = tan 120° = $-\sqrt{3}$

$\therefore$ Equation of OA is $y = \sqrt{3}x$ i.e. $\sqrt{3}\,x - y = 0$ and equation of OB is $y = -\sqrt{3}\,x$ i.e. $\sqrt{3}\,x + y = 0$

Hence required joint equation is

$(\sqrt{3}\,x - y)(\sqrt{3}\,x + y) = 0$ i.e. $3x^2 - y^2 = 0$

141.(A)

Let $r = x\bar{a} + y\bar{b}$

$$\therefore -4\hat{i} - 6\hat{j} - 2\hat{k} = x(-\hat{i} + 4\hat{j} + 3\hat{k}) + y(-8\hat{i} - \hat{j} + 3\hat{k})$$

$$= (-x - 8y)\hat{i} + (4x - y)\hat{j} + (3x + 3y)\hat{k}$$

$\therefore -x - 8y = -4$ ...(1)

$4x - y = -6$ ...(2)

$3x + 3y = -2$ ...(3)

Solving (1) and (2), we get

$y = \frac{2}{3}$ and $x = \frac{-4}{3}$

142.(C)

$(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & -3+0 \\ -6-1 & 9+2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$$

143.(D)

Number of ways of getting doublet = 6

Number of ways of getting a total of 10 are ⇒ (4, 6), (5, 5), (6, 4) i.e. 3 ways

Here (5, 5) is common.

∴ Total ways of getting doublet or total of 10 are 6 + 3 − 1 = 8

Hence required probability = $\frac{36-8}{36} = \frac{28}{36} = \frac{7}{9}$

144.(B)

We have $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = kP$

$\therefore \int \frac{dP}{P} = \int k\,dt \Rightarrow \log P = kt + c$

From given data, we write

$\log 20 = k(0) + c \Rightarrow c = \log 20$

$\therefore \log P = kt + \log 20$

Also $\log 40 = 30k + \log 20$

$\therefore \log 40 - \log 20 = 30k \quad \Rightarrow k = \frac{1}{30}\log 2$

$\therefore \log P = \left(\frac{\log 2}{30}\right)t + \log 20$

When t = 30 + 15 = 45,

$\therefore \log P = \left(\frac{\log 2}{30}\right)(45) + \log 20 = (\log 2)\left(\frac{3}{2}\right) + \log 20$

$= \log(2)^{\frac{3}{2}} + \log 20 = \log\left(2\sqrt{2} \times 20\right)$

$\therefore \quad P = 40\sqrt{2}$ lakhs

145.(D)

We find that in R_4, f(a) = 1 and f(a) = 2. Hence R_4 is not a function.

146.(B)

We have $\cos x = \frac{24}{25} \Rightarrow \sin x = \frac{7}{25}$ … [∵ x lies in 1st quadrant]

Also $\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 = 1 + \sin x = 1 + \frac{7}{25} = \frac{32}{25}$

$\therefore \sin\frac{x}{2} + \cos\frac{x}{2} = \sqrt{\frac{32}{25}} = \frac{4\sqrt{2}}{5} = \frac{8}{5\sqrt{2}}$

147.(B)

$$a+\frac{b}{\log 2} = \int_2^e \left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right]dx$$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = e^t\,dt$

When $x = e$, $t = 1$ and when $x = 2$, $t = \log 2$

$$\therefore\ a+\frac{b}{\log 2} = \int_{\log 2}^{1}\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t\cdot dt = \left[e^t\cdot\frac{1}{t}\right]_{\log 2}^{1} = e-\frac{e^{\log 2}}{\log 2} = e-\frac{2}{\log 2}$$

$\therefore\ a = e, b = -2$

148.(B)

We have lines $y-2=0$ and $4x-3z+5=0$

$$\therefore\ 4x = 3z-5 = 3\left[z-\left(\frac{5}{3}\right)\right]$$

$$\therefore\ \frac{4x}{12} = \frac{3\left[z-\left(\frac{5}{3}\right)\right]}{12} \Rightarrow \frac{x}{3} = \frac{z-\left(\frac{5}{3}\right)}{4},\ y=2$$

Thus line passes through the point $\left(0, 2, \frac{5}{3}\right)$ i.e. a point having position vector $2j+\frac{5}{3}\hat{k}$

Also direction ratios of a line are 3, 0, 4.

Hence required vector equation is

$$\bar{r} = \left(2\hat{j}+\frac{5}{3}\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{k}\right)$$

149.(C)

Let $I = \int \frac{2x^2-1}{x^4-x^2-20}dx$

If $x^2 = t$, then $\frac{2x^2-1}{x^4-x^2-20} = \frac{2t-1}{t^2-t-20}$

Let $\frac{2t-1}{(t-5)(t+4)} = \frac{A}{(t-5)}+\frac{B}{(t+4)}$

$\therefore\ 2t-1 = (t+4)A+(t-5)B$

$\therefore\ 2 = A+B$ and $-1 = 4A-5B$

Solving, we get $B = 1$, $A = 1$

$$\therefore\ I = \int\left[\frac{1}{x^2-5}+\frac{1}{x^2+4}\right]dx = \frac{1}{2\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

150.(A)

$x = a(t+\sin t)$ and $y = a(1-\cos t)$

$\therefore\ \frac{dx}{dt} = a(1+\cos t)$ and $\frac{dy}{dt} = a(\sin t)$

$$\therefore\ \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{a\sin t}{a(1+\cos t)} = \frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{2\cos^2\frac{t}{2}} = \tan\frac{t}{2}$$